Establishing the Complex Measurement Ability of a Homodyne Network Analyzer via Self-Calibration

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ABSTRACT

A homodyne phase shifter controlled double-reflectometer is presented. The ability for complex measurement, i.e. the knowledge of the phase shifter characteristics, is established with fully unknown standards by only exploiting reciprocity. If a system error correction is performed the data needed for error correction contain enough information to determine the phase shifter behavior. Therefore no additional standards are needed.

INTRODUCTION

Network analyzers are in widespread use. Mostly they are based on the well-known heterodyne concept in which the rf-signal is down converted into an if-signal to measure the complex information, i.e. magnitude and phase. Nevertheless there exist other ways to realize such an equipment. For example the six-port network analyzer [3], which uses power detectors or the so-called homodyne network analyzer [1] which uses a coherent detection. In contrast to the power detectors the coherent detection is a linear detection, thus providing a higher dynamic range. Unfortunately the output signal is not proportional to the complex but rather to the real part of the rf-information. A further measurement is necessary to get the information about the imaginary part, demanding for a 90°-phase shifter. Phase shifts φ different from 90° are possible, but they must also exactly be known. Although the homodyne concept is very simple and uses an inexpensive rf-part the lack of phase shifters with an a priori known phase shift is one reason that homodyne concepts are not in use for commercially available laboratory measurement equipments. Therefore it has been investigated, if it is possible to determine the effective phase shift of the imperfect device in situ. With a procedure described below the homodyne detector is indeed able to measure the complex information of the rf-signal.

THEORY

General Description

Therefore a set-up is described which is capable to take four complex measurements, see Fig. 1. Firstly we treat four-port 1 depicted at the left side. It is introduced to provide a measure of the wave propagating towards the device under test (DUT) as well as a measure of the wave emerging from the DUT. For example b_{4A} should be a measure of the incident wave and b_{2A} a measure of the reflected wave. Due to imperfections like a finite directivity or mismatched ports both measurements are disturbed.

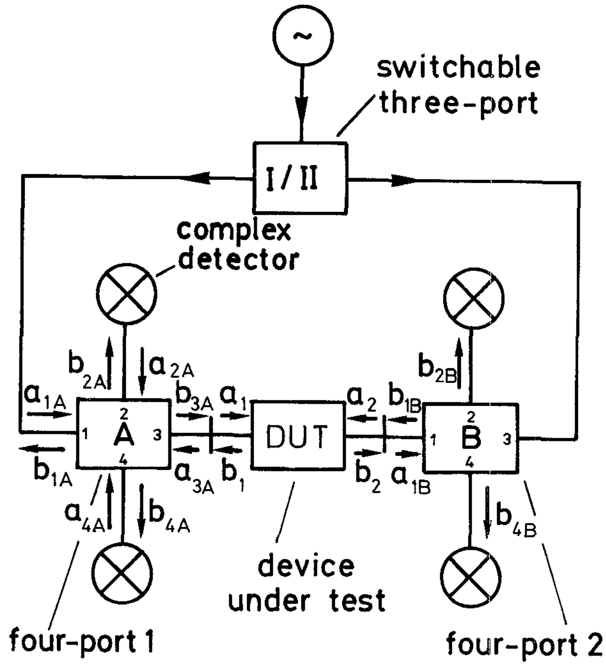

Fig. 1: The principal block diagram of the set-up

Thus in our discussion any interior of the four-port is allowed, if it is ensured that the readings of b_{2A} and b_{4A} are independent from each other.

Now we introduce a fictive four-port which includes the four-port 1 and the mismatches and losses of the detectors connected. Therefore $a_{2A} = a_{4A} = 0$ and it holds

$$b_{1A} = S_{11A}a_{1A} + S_{13A}a_{3A} \quad (1)$$
$$b_{2A} = S_{21A}a_{1A} + S_{23A}a_{3A} \quad (2)$$
$$b_{3A} = S_{31A}a_{1A} + S_{33A}a_{3A} \quad (3)$$
$$b_{4A} = S_{41A}a_{1A} + S_{43A}a_{3A}\ . \quad (4)$$

These four equations can be reduced in a straight forward way to the relationship

$$\begin{pmatrix} b_{2A} \\ b_{4A} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} a_{3A} \\ b_{3A} \end{pmatrix}. \quad (5)$$

In a similar approach four-port 2 is treated yielding

$$\begin{pmatrix} b_{2B} \\ b_{4B} \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \begin{pmatrix} b_{1B} \\ a_{1B} \end{pmatrix}. \quad (6)$$

For any two-port, i.e. a DUT or a calibration standard, connected to the measurement ports

$$\begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} \tag{7}$$

holds. Under use of the boundary conditions

$$b_1 = a_{3A},\ a_1 = b_{3A},\ a_2 = b_{1B} \text{ und } b_2 = a_{1B} \tag{8}$$

the equation

$$\begin{pmatrix} b_{2A} \\ b_{4A} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}^{-1} \begin{pmatrix} b_{2B} \\ b_{4B} \end{pmatrix}$$

$$= \mathbf{A}\ \mathbf{T}\ \mathbf{B}^{-1} \begin{pmatrix} b_{2B} \\ b_{4B} \end{pmatrix} \tag{9}$$

is derived. In order to provide a second vector equation of this type the three-port (Fig. 1) is turned to its second position, position II. This might be the second position of a microwave switch, but any alteration of its signal splitting behavior is sufficient. However, a microwave switch may be a prefered realization. The readings in the second state of the three-port are indicated by the prime and fit

$$\begin{pmatrix} b'_{2A} \\ b'_{4A} \end{pmatrix} = \mathbf{A}\ \mathbf{T}\ \mathbf{B}^{-1} \begin{pmatrix} b'_{2B} \\ b'_{4B} \end{pmatrix}. \tag{10}$$

These two vector equations, eqn. 9 and eqn. 10, are combined to the matrix equation

$$\begin{pmatrix} b_{2A} & b'_{2A} \\ b_{4A} & b'_{4A} \end{pmatrix} = \mathbf{A}\ \mathbf{T}\ \mathbf{B}^{-1} \begin{pmatrix} b_{2B} & b'_{2B} \\ b_{4B} & b'_{4B} \end{pmatrix}, \tag{11}$$

which is finaly denoted as

$$\mathbf{A}\ \mathbf{T}\ \mathbf{B}^{-1} = \begin{pmatrix} b_{2A} & b'_{2A} \\ b_{4A} & b'_{4A} \end{pmatrix} \begin{pmatrix} b_{2B} & b'_{2B} \\ b_{4B} & b'_{4B} \end{pmatrix}^{-1} \overset{def}{=} \mathbf{M}. \tag{12}$$

This description is usually found in network analyzer calibration theory at the starting point. The matrices $\mathbf{A}$ and $\mathbf{B}^{-1}$ are the well-known error-matrices in order to consider the imperfections of the set-up.

If it is possible to provide the measurement matrix $\mathbf{M}$ even with a homodyne set-up, one is able to proceed with any calibration procedure relying on a formalism similar to eqn. 12. This might be for example the TSD-procedure [2], the TRL-procedure [3], the TMR or TAN-procedure [4].

Establishing The Complex Measurement Ability

In this section the complex measurement ability is established without an additional expense of calibration standards. This means that either no further standards or only fully unknown standards are required.

Therefore the behavior of one of the coherent detectors will be examined a little further (Fig. 2), e.g. the detection of b_{A2}. If the phase shifter in the path of the local oscillator is switched off, the detected voltage will be denoted as U_{2A} and if it is turned on, the same quantity will be $\tilde{U}_{2A}$. It can easily be seen that these two voltages can be assembled to reconstruct the complex wave b_{2A} by

$$b_{2A} = b_{2A}(p) = \alpha_{2A}(U_{2A} + p\,\tilde{U}_{2A}), \overset{def}{=} \alpha_{2A}U_{2A}(p)\ , \tag{13}$$

where α_{2A} is a proportionality factor. At this time the 'weighting factor' p is still unknown. For example, if the phase shifter is an ideal one, i.e. a minus-90°-phase shifter, p equals j and therefore $U_{2A}(p) = U_{2A} + j\,\tilde{U}_{2A}$.

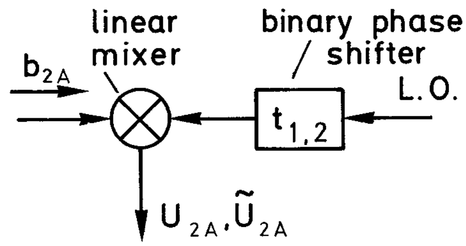

Fig. 2: A homodyne detector with an effective binary phase shift $t_{1,2} = |t_{1,2}|e^{j\varphi_{1,2}}$

However, p has to be determined by some kind of calibration procedure. Therefore eqn. 13 and similar equations for the other complex waves will be substituted into eqn. 11 leading to

$$\begin{pmatrix} U_{2A}(p) & U'_{2A}(p) \\ U_{4A}(p) & U'_{4A}(p) \end{pmatrix} = \mathbf{A}\ \mathbf{T}\ \mathbf{B}^{-1} \begin{pmatrix} U_{2B}(p) & U'_{2B}(p) \\ U_{4B}(p) & U'_{4B}(p) \end{pmatrix} \tag{14}$$

$$\longrightarrow \quad \mathbf{M}_A(p) = \mathbf{A}\ \mathbf{T}\ \mathbf{B}^{-1}\ \mathbf{M}_B(p)$$

$$\longrightarrow \quad \mathbf{A}\ \mathbf{T}\ \mathbf{B}^{-1} = \mathbf{M}_A(p)\ \mathbf{M}_B(p)^{-1} = \mathbf{M}(p), \tag{15}$$

in which the proportionality factors α_i are included into the error matrices $\mathbf{A}$ and $\mathbf{B}^{-1}$. As the phase shifter is in the common path of the mixers, see Fig. 3, the factor p is always the same. This assumption is not always valid. But if the phase shifter only shows a small parasitic amplitude modulation and if the mixers are fairly equal the error is of higher order small. Nevertheless, more general solutions are also available in which each detector may have another weighting factor, i.e. p_1, p_2, p_3, p_4. However, here only the 'simple-p-case' is treated, the more general solution is reserved to the extended version of this paper in [6].

In order to determine the weighting factor p two completely unknown calibration two-ports with the wave transmission matrices $\mathbf{N1}$ and $\mathbf{N2}$ are connected to the measurement ports, leading to the measurement matrices which are functions of p

$$\mathbf{M1}(p) = \mathbf{A}\ \mathbf{N1}\ \mathbf{B}^{-1}, \tag{16}$$

$$\mathbf{M2}(p) = \mathbf{A}\ \mathbf{N2}\ \mathbf{B}^{-1}. \tag{17}$$

Taking the inverse of $\mathbf{M1}(p)$ and multiplying by $\mathbf{M2}(p)$ the new matrix is denoted as

$$\mathbf{Q}(p) = \mathbf{M2}(p)\ \mathbf{M1}(p)^{-1} \tag{18}$$

$$= \left(\mathbf{M2}_A(p)\ \mathbf{M2}_B(p)^{-1}\right) \left(\mathbf{M1}_A(p)\ \mathbf{M1}_B(p)^{-1}\right)^{-1}$$

with

$$\det \mathbf{Q}(p) = \frac{\det \mathbf{M2}_A(p) \ \det \mathbf{M1}_B(p)}{\det \mathbf{M2}_B(p) \ \det \mathbf{M1}_A(p)}. \qquad (19)$$

On the other hand it holds

$$\det \mathbf{Q}(p) = \det \left(\mathbf{M2}\ \mathbf{M1}^{-1}\right) \qquad (20)$$
$$= \det \left(\mathbf{A}\ \mathbf{N2}\ \mathbf{B}^{-1} \left(\mathbf{A}\ \mathbf{N1}\ \mathbf{B}^{-1}\right)^{-1}\right) = \frac{\det \mathbf{N2}}{\det \mathbf{N1}}.$$

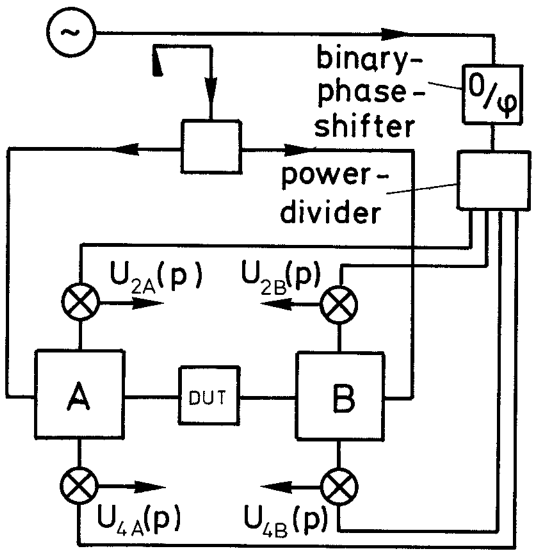

Fig. 3: The homodyne double-reflectometer

If **N1** and **N2** are the transmission matrices of reciprocal but otherwise unknown two-ports their determinants equal one,

$$\det \mathbf{N1} = \det \mathbf{N2} = 1, \qquad (21)$$

and therefore

$$\det \mathbf{M2}_A(p)\ \det \mathbf{M1}_B(p)\ -\ \det \mathbf{M2}_B(p)\ \det \mathbf{M1}_A(p) \overset{!}{=} 0 \qquad (22)$$

holds. Further algebraic treatment leads to a polynomial characteristic equation of fourth degree

$$a_4\, p^4 + a_3\, p^3 + a_2\, p^2 + a_1\, p + a_0 = 0 \qquad (23)$$

which must be fullfilled by the weighting factor p. The coefficients a_i in eqn. 23 are real because they have been assembled by using measured real voltages only.

The characteristic equation 23 can be solved by various methods. Good results have been obtained using Müller's method [5].

However, a linear solution is also possible, for example to provide starting values for the non-linear solution. At the expense of one more unknown standard the linear approach can be applied.

For this purpose a third reciprocal two-port is connected and measured leading to $\mathbf{M3}_A(p)$ and $\mathbf{M3}_B(p)$. These new matrices are combined with the existing ones leading to two further equations

$$\det \mathbf{M3}_A(p)\ \det \mathbf{M1}_B(p)\ - \det \mathbf{M3}_B(p)\ \det \mathbf{M1}_A(p) \overset{!}{=} 0, \qquad (24)$$

$$\det \mathbf{M3}_A(p)\ \det \mathbf{M2}_B(p)\ - \det \mathbf{M3}_B(p)\ \det \mathbf{M2}_A(p) \overset{!}{=} 0, \qquad (25)$$

and finally to a system of characteristic equations.

$$a_{14}\, p^4 + a_{13}\, p^3 + a_{12}\, p^2 + a_{11}\, p + a_{10} = 0 \qquad (26)$$
$$a_{24}\, p^4 + a_{23}\, p^3 + a_{22}\, p^2 + a_{21}\, p + a_{20} = 0 \qquad (27)$$
$$a_{34}\, p^4 + a_{33}\, p^3 + a_{32}\, p^2 + a_{31}\, p + a_{30} = 0 \qquad (28)$$

By eliminating p^4 and p^3 the system is reduced to

$$\tilde{a}_2\, p^2 + \tilde{a}_1\, p + \tilde{a}_0 = 0 \qquad (29)$$

which has the two solutions p_1 and p_2. Due to the fact that p depends on the phase shift φ which must be different from multiples of 180° the solutions are always complex. As the $\tilde{a}_i$ are real numbers the solution yields a pair of complex conjugates, i.e. $p_2 = p_1^*$. This ambiguity is easily removed by a rough knowledge of the design of the phase shifter, i.e. one has only to decide whether the phase shifter does provide a positive or a negative phase shift. If eqn. 29 has two real roots both are useless and the measurement should be reconsidered. On the other hand in this case it might be possible that the assumption of the simple-p-case is not valid. However, correct p-values are always complex.

DISCUSSION

It has been investigated how the procedure works, if the input data are not exact and how the performance drops with increasing measurement errors.

There are in general two opposite cases to be regarded which both are in practical use.

In the first case each detection channel gets information about both the wave leaving the measurement port and the wave entering it. Obviously the superposition must be different in the two measurement channels of each of the reflectometers. An example for this principle is the six port reflectometer.

In the second case, i.e. the ordinary heterodyne four port reflectometer, a considerable effort is made to separate the two waves mentioned above. The mixture of information which is of essential importance for the six port reflectometer is regarded as an error.

The theoretical approach presented above provides no restrictions in generality. Thus the more general case of arbitrary superposition is permitted. By regarding the second case it should be stressed that two measurements, e.g. $U_{4A}(p)$ and $U_{4B}(p)$, are decoupled from the measurement ports. Hence eqn. 24 and eqn. 25 do not provide any further information and the linear approach fails, but the non-linear approach using eqn. 23 still works.

The linear solution starts to work if by some imperfections signals entering the measurement ports are coupled to $U_{4A}(p)$ and $U_{4B}(p)$. Therefore it is concluded that for any practical set-up the linear approach will nearly always work.

However, the linear approach should be omitted, because the error sensivity increases rapidly if the system becomes more and more ideal in the sence of separating the two waves leaving and entering the measurement ports.

But also in the case of arbitrary superposition the linear approach proves to be more susceptible to data errors. Therefore it is investigated how the calculated weighting factor differs from the ideal one if the input data become noisy. The simulations are performed under practical assumptions like losses, mismatches, finite directivities, switching dependend mismatches and cross talk, etc.

The weighting factor is assumed to be $p = j = e^{j90^\circ}$. The source provides 3 dBm of microwave power and the noise level in the measurement channels is assumed to be variable.

In Fig. 4a the standard error of the weighting factor magnitude is plotted as a function of the noise level. It turns out that the linear approach requires a signal to noise ratio which is about 35 dB better than the direct solution of the non-linear characteristic equation. The standard error of the angle of the reconstructed weighting factor shows a similar behavior, Fig. 4b.

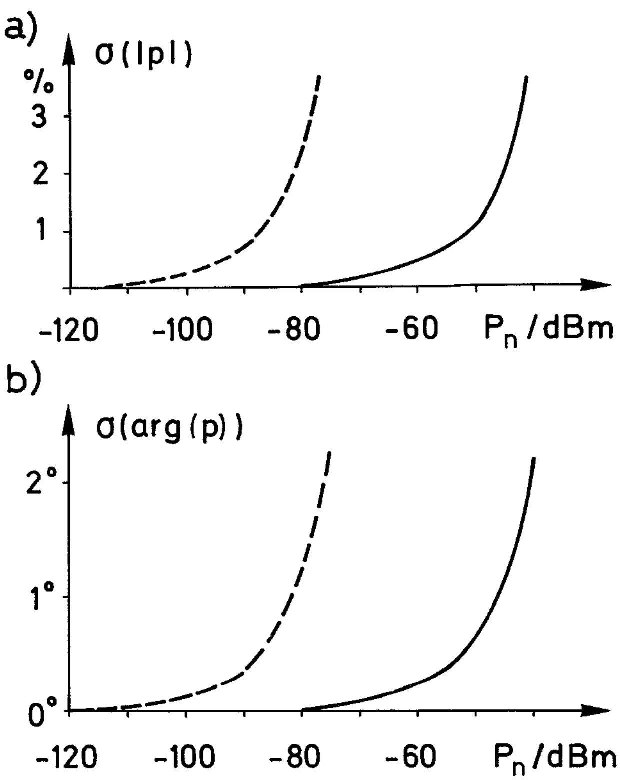

Fig. 4 a: Standard error $\sigma(|p|)$ of the magnitude of the weighting factor p versus noise level P_n
b: Standard error $\sigma(\arg(p))$ of the argument of p versus noise level
— direct non-linear solution
- - linear solution

The comparatively poor behavior of the linear approach has the consequence that it can at best serve as a starting value. But the non-linear solution has proved to be reliable even with arbitrary initial values.

However, the simulations show that the standard error of both the weighting factor magnitude and its argument remain for practical noise levels better than 0.04% and 0.02° which is well below any measurement accuracy. This means that measurement errors due to thermal noise or quantisation noise are not able to influence the determination of the weighting factor noticeably.

CONCLUSION

Via the measurement of two *arbitrary* and *unknown* but *reciprocal* networks, it is possible to determine the complex weighting factor p, i.e. to establish the ability of measuring complex information in a homodyne network analyzer. For example this can be done by using a sliding line of arbitrary characteristic impedance and unknown length and arbitrary reflections.

In order to reduce the effort of connecting calibration standards it is possible to use the data needed anyway to calibrate the set-up for system error removal. Calibration procedures like TSD [2], TRL [3], TAN or TMR [4] provide well conditioned characteristic equations. Therefore it is possible to establish the ability of complex measurements without any additional expense and to proceed as in a normal network analyzer calibration.

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REFERENCES

[1] KING, R.J., Microwave Homodyne Systems, Peter Peregrinus, 1978

[2] FRANZEN, N.R., SPECIALE, R.A., A New Procedure for System Calibration and Error Removal in Automated S-Parameter Measurements, Proc. 5. EuMC, Hamburg, 1975, pp. 69 - 73

[3] ENGEN, G.F., HOER, C.A., Thru-Reflect-Line: An Improved Technique for Calibrating the Dual Six Port Automatic Network Analyzer, IEEE MTT-27, Dec. 1979, pp. 987 - 993

[4] EUL, H.J., SCHIEK, B., Thru-Match-Reflect: One Result of a Rigorous Theory for De-embedding and Networkanalyzer Calibration, Proceedings of the 18th European Microwave Conference, Stockholm, 1988, pp. 909 - 914

[5] SCHWARZ, H.R., Numerische Mathematik, Teubner Verlag, Stuttgart 1986

[6] EUL, H.J., Self-Calibration Concepts for Establishing the Complex Measurement Ability of Homodyne Network Analyzers, submitted for publication in the IEEE MTT Symposium Issue, Dec. 1989